142_12c_5.4

Thursday, February 27, 2020 10:53 AM

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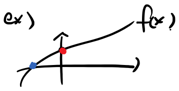
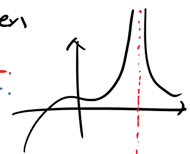
5.4 Supplement: Additional Curve Sketching

Graphing Strategy $f(x)$. Objective: Draw $f(x)$ approximately.

1. Analyze $f(x)$. ex)
   - Find the domain of f.
   - Find the intercepts. ⟹ y-intercepter. x-intercepter.
   - Find the asymptotes. Horizontal asymptote. Vertical asymptote.



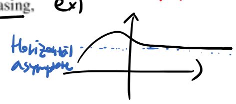
ex)

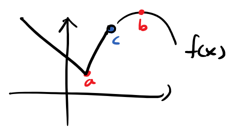


Vertical asymptote

2. Analyze $f'(x)$. inc/dec
   - Find the partition numbers of $f'(x)$. (Critical Value) $f'(x)=0$ or DNE.
   - Find the critical values of f. ⟹ $f'(x)=0$, or DNE and $f(x)$ is defined.
   - Construct a sign chart for $f'(x)$ to determine where f is increasing/decreasing, ex) and find any local/relative extrema of f.

	a		c		b	
$f'(x)$	−	+		+		−





a c b : partion numbers (point)

$f'(x)$ DNE, $f'(x)=0$

a b : Critical Value (since $f(x)$ is defined at a, b.)

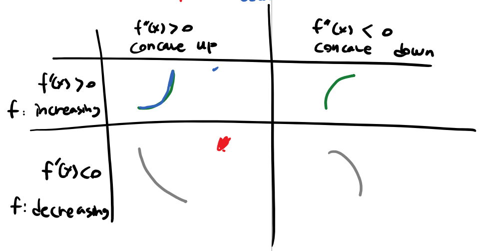
3. Analyze $f''(x)$.
   - Find the partition numbers of $f''(x)$.
   - Construct a sign chart for $f''(x)$ to determine where f is concave upward/downward, and find any inflection points.

ex)

$f''(x)>0$ Concave up | $f''(x)<0$ Concave down

4. Sketch the graph of f. Create a **combined number line** that shows the "shapes" of the graph, then draw the graph using all the pertinent information you found in steps 1-3.

Example: Use the graphing strategy to analyze the function $f(x) = \dfrac{2x^2+11x+14}{x^2-4}$. State all pertinent information and sketch the graph of f.



Example: Use the given information below to sketch the graph of f. **You must include sign charts with pertinent information for both $f'(x)$ and $f''(x)$, as well as a combined number line.**

partition #. (Critical value + f is not defined pts)

ex)

Sign of $f'(x)$ + − +

↖ Sign chart! for $f'(x)$

- $\boxed{\text{Domain of } f}$: $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$
- ① $f(-2)=1$, $f(0)=0$, and $f(2)=1$
- $f'(x)>0$ on $(-\infty,-1)$ and $(0,1)$
- $f'(x)<0$ on $(-1,0)$ and $(1,\infty)$
- $f''(x)>0$ on $(-\infty,-1)$, $(-1,1)$, and $(1,\infty)$
- ② Vertical asymptotes at $x=-1$ and $x=1$
- ③ $\lim_{x\to\infty} f(x)=0$ and $\lim_{x\to-\infty} f(x)=0$

$(-\infty,-1)$ $(-1,0)$ $(0,1)$ $(1,\infty)$

−1 0 1

Sign of $f'(x)$ + − + −

⇒ There are three partition #s, : −1, 0, 1

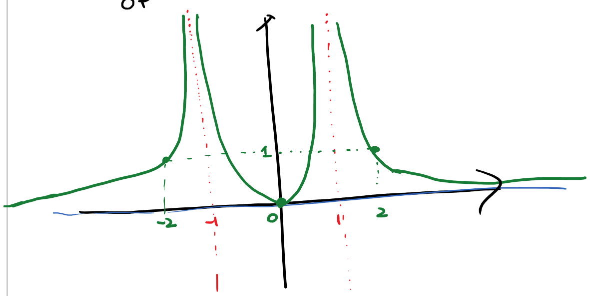
Critical value: 0.

−1 1

Sign of $f''(x)$ + + +

⇒ There are two partition numbers −1, 1.

Inflection point! No Inflection pts



Def: Partition # of $f'(x)$ (or $f''(x)$) is x values (numbers) s.t.

$f'(x)=0$ or DNE. or ($f''(x)=0$ or DNE)

Def: Critical Value of $f(x)$ is the partion number of $f(x)$ s.t. $f(x)$ is defined.

Def

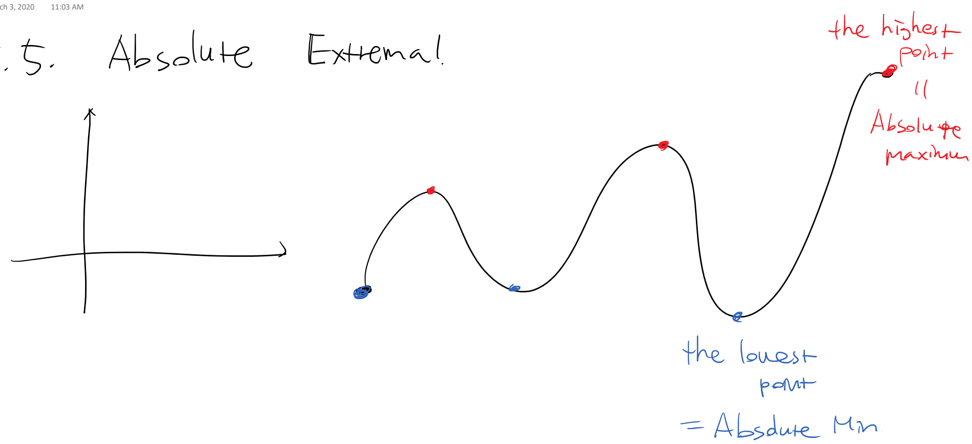
Inflection point is a partition # of $f''(x)$ s.t.

① $f(x)$ is defined

② Sign changed between the number.

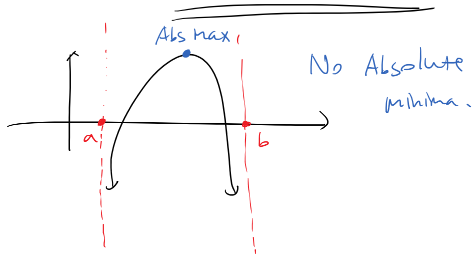
Practice: On your on paper, try using the graphing strategy to analyze the function $f(x)=\dfrac{\ln x}{x}$. State all pertinent information and sketch the graph of f.

5.5. Absolute Extrema!



* Absolute Max / Min is defined on

"closed interval" $[a, b]$ closed (a, b) open



* Find Absolute Max / Min. on $[a, b]$

① Do the second derivative test to find local extremas.

② Calculate $f(a)$, $f(b)$.

③ Compare all function values in ①, ② to determine abs max or abs min!

Ex) $f(x) = x^3 - 2x + \frac{1}{x}$ $= x^{-1}$ when $x \in [-3, 10]$. (a b)

Find Abs max / min. domain $x \neq 0$

① Find local max / min using second derivative test!

$$f'(x) = 3x^2 - 2 - \frac{1}{x^2} \quad -x^{-2}$$

$$f''(x) = 6x + \frac{2}{x^3} \quad + 2 \cdot x^{-3}$$

Critical values! $f'(x) = 0$ or DNE + $f(x)$ is defined.

$$f'(x) = 0 \Rightarrow 3x^2 - 2 - \frac{1}{x^2} = 0$$

$$3x^4 - 2x^2 - 1 = 0 \qquad x^2 = u$$

$f'(x) = 0 \Rightarrow 3x^2 - 2 - \frac{1}{x^2} = 0$

$$3x^4 - 2x^2 - 1 = 0 \qquad x^2 = u$$

$$3u^2 - 2u - 1 = 0$$

$1 \quad -1$

$3 \quad 1$

$$(x^2 - 1)(3x^2 + 1) = 0$$

$$(x-1)(x+1)(3x^2+1) = 0$$

$f'(x) = 0$ when $x = 1, -1$.

$f'(x) = \text{DNE}$ when $x = 0$.

Critical values : $-1, 1$.

$$f''(-1) = 6\cdot(-1) + \frac{2}{(-1)^3} = -6 - 2 = -8 < 0$$

$$f''(1) = 6\cdot 1 + \frac{2}{1} = 8 > 0$$

By second derivative test,

$f''(-1) < 0 \Rightarrow f(-1) =$ local maxima

$f''(1) > 0 \Rightarrow f(1) =$ local min.

$$f(1) = 1^3 - 2\cdot 1 + \frac{1}{1} = 1 - 2 + 1 = 0$$

$$f(-1) = (-1)^3 - 2\cdot(-1) + \frac{1}{-1} = -1 + 2 - 1 = 0$$

(2) Find $f(-3) = (-3)^3 - 2\cdot(-3) + \frac{1}{-3} = -27 + 6 - \frac{1}{3} = -21 - \frac{1}{3}$

$f(10) = 10^3 - 2\cdot 10 + \frac{1}{10} = 1000 - 20 + \frac{1}{10} = 980.01$

(3) Compare all the values

from (1) $\quad f(1) = 0$

$\qquad\qquad f(-1) = 0$

from (2) $\quad f(-3) = -21 - \frac{1}{3}$ the lowest!

$\qquad\qquad f(10) = 980.01$ the highest

$f(-3)$ is the absolute min of $f(x)$

$f(10)$ is absolute max of $f(x)$

$$f(x) = x^4 + 2x^3 + 3x^2 + 2x + 1 \qquad [0, 5]$$

$5^4 \quad 2\cdot 5^3 \quad + 3\cdot 5^2 \quad + 10 + 1$

Absolute max / min on $[0, 5]$

① Second derivative test.

$$f'(x) = 4x^3 + 6x^2 + 6x + 2$$

$$f''(x) = 12x^2 + 12x + 6$$

$\boxed{f'(x) = 0}$ $\quad 2(\underbrace{2x^3 + 3x^2 + 3x + 1}_{> 0 \text{ on } [0,5]}) = 0$

$\Rightarrow$ there is no critical value from $f'(x) = 0$.

$\boxed{f'(x)\ \text{DNE}}$ $\Rightarrow$ no local max on min.

② $f(0)$ $f(5)$

$$f(0) = 0 + 0 + 0 + 0 + 1 = 1$$

$$f(5) = 5^4 + 2\cdot 5^3 + 3\cdot 5^2 + 2\cdot 5 + 1 > 1$$

③ Compare all these values

from ① Nothing.

from ② $f(0) = 1$ $\quad f(0)$: abs min

$f(5) > 1$ $\quad f(5)$ " max.