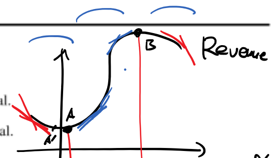
142_12c_5.1

Thursday, February 20, 2020 11:04 AM

142_12c_5.1

5.1 Supplement: The First Derivative

Increasing and Decreasing Functions



- If $f'(x) > 0$ on an interval, then f is increasing on that interval.
- If $f'(x) < 0$ on an interval, then f is decreasing on that interval.

Some examples include...

Critical value = when $f'(x) = 0$ or DNE

Sign chart.

$x < A'$ ⇒ $f'(x) < 0$ ⇒ slope tangent line is negative.

Sign of $f'(x)$

$A' < x < B'$ ⇒ $f'(x) > 0$ ⇒ the slope of tangent line increases.

x A' x B'

$(-)$ $(+)$ $(-)$

sign chart

*To determine where f is increasing/decreasing, we will create a sign chart. What "important" x values should we include on our sign chart? What do we call these x values?

x values which gives you "critical point"

Example: Find where $f(x) = (1-x)^{1/3}$ is increasing/decreasing.

① $f'(x) = -\frac{1}{3}(1-x)^{-\frac{2}{3}}$ chain rule

$f(x) = u^{\frac{1}{3}}$ $u = 1-x$

$f'(x) = \frac{1}{3}u^{-\frac{2}{3}} \cdot u'$

$= \frac{1}{3}(1-x)^{-\frac{2}{3}} \cdot (-1)$

$= -\frac{1}{3}(1-x)^{-\frac{2}{3}}$

② Critical points!

$f'(x) = 0$ or DNE.

$f'(x) = -\frac{1}{3} \cdot (1-x)^{-\frac{2}{3}}$

$= -\frac{1}{3} \cdot \frac{1}{(1-x)^{\frac{2}{3}}}$ ⇒

$(1-x)^{\frac{2}{3}} = ((1-x)^{\frac{1}{3}})^2$

1) when $f'(x)$ is not defined,

when $x = 1$, then $f'(x)$ is not defined.

2) $f'(x) = 0$ (There is no x making $f'(x) = 0$

③ Find sign chart

when $x < 1$

$x < 1 \Rightarrow 1-x > 0$

$f'(x) < 0$

when $x > 1 \Rightarrow 1-x < 0$

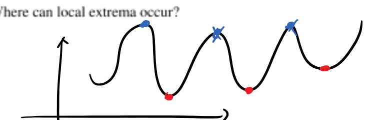
Sign of $f'(x)$ $(-)$ 1 $(-)$

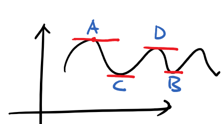
④ when $x \neq 1$, $f(x)$ is decreasing. from sign chart.

If $x \in (-\infty, 1) \cup (1, \infty)$, "

Local Maximum and Minimum Values:

- A function f has a local maximum at c if $f(c) \geq f(x)$ when x is near c.
- A function f has a local minima at c if $f(c) \leq f(x)$ when x is near c.
- The local maximum and and minimum values of f are called the local extrema. of f.
- Where can local extrema occur?



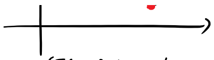


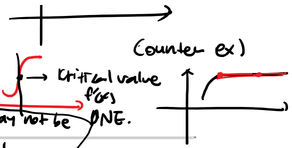
Counter ex)

(If f has local extrema at x, then x is critical value.)

critical value $f'(x)$

Critical Values:

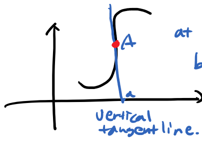




Critical Values: If f has local extrema at x, then x is critical value. But, if f has critical value at x, it may not be the local extrema.

critical value $f'(x)$ DNE.

- If f has a local max/min at c, then
- A **critical value** of a function f is a number c in the domain of f such that either $f'(c) = 0$ or $f'(c)$ does not exist. DNE.
- If f has a local max/min at c, then c is critical value.
- Is the converse of the above statement true? False



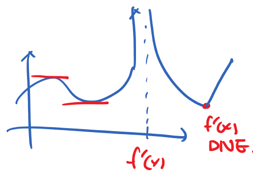
at A, $f'(x)$ is DNE but f doesn't have local extrema at $x=a$.

Question: How do we find the critical values of a function f?

① If f is given by graph,

1) Local extrema by intuition.

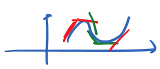
2) $f'(x)$ does not exists.



② If f = equation,

1) find $f'(x)$

2) Figure out when $f'(x)=0$ or DNE algebraically.



The First Derivative Test - Suppose that c is a number in the interval (a,b) and f is continuous on the interval (a,b). Also, let c be a critical value of f. $f'(c)=0$ or DNE.

- If f' changes from positive to negative at c, then f has a local maximum at c.
- If f' changes from negative to positive at c, then f has a local minimum at c.
- If f' does not change sign at c, then f has no local maximum or minimum at c.

Example: For each of the following, determine where the function is increasing/decreasing and find any local extrema.

a) $f(x) = x^3 - 6x^2 + 9x + 1$

① $f'(x) = 3x^2 - 12x + 9 = 3(x^2 - 4x + 3) = 3(x-1)(x-3)$

② Critical values

1) $f'(x)$ is DNE? No x make $f'(x)$ DNE.

2) $f'(x) = 0$? $x = 1, 3$.

All critical values are $1, 3$

③ Sign chart!

Sign of $f'(x)$: x (+) | 1 | x (−) | 3 | x (+)

$f'(x) = 3(x-1)(x-3)$ (−)(−)

$x<1 \Rightarrow x-1<0$

$x<1 \Rightarrow x<3 \Rightarrow x-3<0$

$f'(x) = 3(x-1)(x-3)$ (+)(−)

$f'(x) = 3(x-1)(x-3)$ (+)(+)

④ Conclusion.

$f'(x)>0$ when $x \in (-\infty, 1) \cup (3, \infty)$

$f'(x)<0$ when $x \in (1, 3)$

$\Rightarrow$ $f(x)$ is increasing $x \in (-\infty, 1) \cup (3, \infty)$

$f(x)$ is decreasing $x \in (1, 3)$

⑤ Local extrema.

Local max $= (x=1)$

Local min. $x=3$

b) $f(x) = \frac{1}{x-2}$

$= (x-2)^{-1}$

$u = x-2$

$= u^{-1}$

$u' =$

① $f'(x) = -1 \cdot u^{-2} \cdot u' = -1 \cdot (x-2)^{-2} \cdot 1 = -\frac{1}{(x-2)^2}$

chain rule

② Critical values

1) $f'(x)$ DNE? when bottom $=0 \Rightarrow \boxed{x=2.}$

2) $f'(x) = 0$? Never!

③ Sign chart!

Sign of $f'(x)$: x (−) | 2 | x (−)

④ Conclusion

f is decreasing when $x \neq 2$.

$\Leftrightarrow x \in (-\infty, 2) \cup (2, \infty)$

⑤ Local extrema

No local

Sign of f'(x): x $(-)$ 2 $=$ x $(-)$

$\Leftrightarrow x \in (-\infty, 2) \cup (2, \infty)$

⑤ Local extrema
No local extrema!

$x < 2$
$\Rightarrow (x-2) < 0$

$f'(x) = -\underbrace{\frac{1}{(x-2)^2}}_{(+)}$ → $(-)$

$x > 2 \Rightarrow (x-2) > 0$

$f'(x) = -\underbrace{\frac{1}{(x-2)^2}}_{(+)}$ → $(-)$

c) $f(x) = 8\ln x - x^2$

① $f'(x) = \frac{8}{x} - 2x$

② Critical Values

1) $f'(x)$ DNE ? when $x = 0$

2) $f'(x) = 0$? when $x = 2, -2$.

$x \cdot \left(\frac{8}{x} - 2x\right) = 0 \cdot x$

$8 - 2x^2 = 0$

$8 = 2x^2$

$4 = x^2$ $x = \pm 2$

③ Sign chart.

x -2 x 0 x 2 x

Sign of $f'(x)$: $(+)$ $(-)$ $(+)$ $(-)$

$f'(x) = \frac{8}{x} - 2x$. $(-)$ $(+)$

$f'(x) = \frac{8}{x} - 2x$ $(-)$ $(+)$

$f'(x) = \frac{8}{x} - 2x$ $(+)$ $(-)$

cannot determine

⇒ plug in -4 plug in $x = -1$ plug in $x = 1$ plug in 4

$f'(-4) = \frac{8}{-4} - 2\cdot(-4)$
$= -2 + 8 = 6 > 0$

$f'(-1) = \frac{8}{-1} - 2(-1)$
$= -8 + 2$
$= -6 < 0$

$f'(1) = \frac{8}{1} - 2\cdot 1$
$= 8 - 2$
$= 6 > 0$

$f'(4) = \frac{8}{4} - 2\cdot 4$
$= 2 - 8$
$= -6 < 0$

④

$f'(x) > 0$ when $x \in (-\infty, -2) \cup (0, 2)$ ⇒ $f(x)$ is increasing when $x \in (-\infty, -2) \cup (0, 2)$

$f'(x) < 0$ when $x \in (-2, 0) \cup (2, \infty)$ ⇒ $f(x)$ is decreasing when $x \in (-2, 0) \cup (2, \infty)$

* local max $x = -2, 2$

* Special Case
At $x = 0$,
$f(x)$ is not defined
⇒ x is not local max or " min

d) $f(x) = (x^2 - 3x - 4)^{4/3}$

e) $f(x) = (x+2)e^x$