Sketching f' from f:

Observe the important points and general behavior of the original graph:

1) Points at which a tangent line is horizontal

when $f'(x) = 0$

2) Intervals over which the graph is increasing or decreasing

$f'(x) > 0 \Rightarrow$ graph is increasing ($f(x)$)

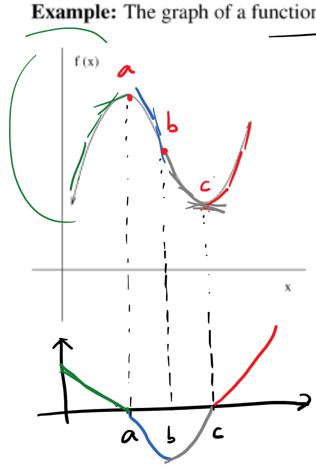
$f'(x) < 0 \Rightarrow f(x)$ is decreasing

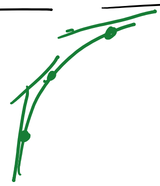
3) Inflection points

These! points at which sign of the slope of tangent line is changed

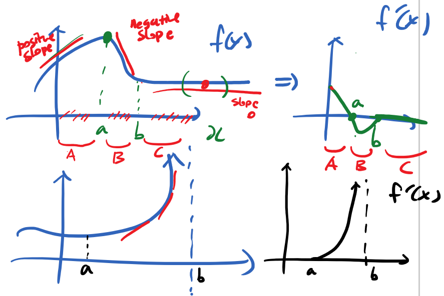
4) Places at which the graph appears to be horizontal or leveling off

Example: The graph of a function f is given below. Sketch the graph of f'.

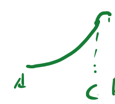




$f'(x)$

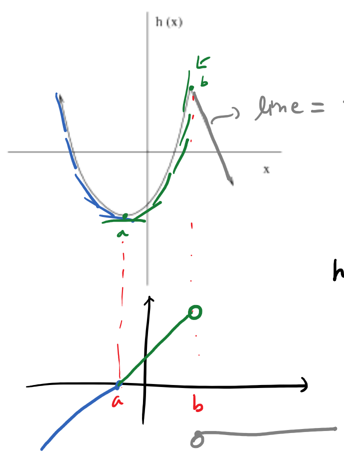


ex)



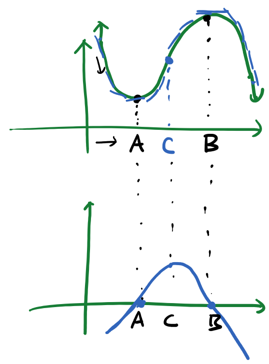
Inflection pt is not symmetrical to A and B

Example: Sketch the derivative of the function shown below.



line = tangent line

$h'(x)$



f'(x) increasing f'(x) decreasing.

A, B $\Rightarrow f'(x) = 0$

C : inflection pt.

① Specify points where $f'(x) = 0$ ex) A, B and inflection pts ex) C

② Draw the graph corresponding to increasing or decreasing status of $f'(x)$

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4.1 Supplement: Derivatives of Powers, Exponents, and Sums

Derivative Notation - If $y = f(x)$, then

$$f'(x) \quad y' \quad \frac{dy}{dx} \quad \frac{d}{dx}f(x)$$

all represent the derivative of f at x.

$f(x)$

$f'(x) = \left(\begin{array}{l} \text{limit (when } h \to 0) \\ \text{difference quotient} \\ \text{between } (x+h, f(x+h)) \\ (x, f(x)) \end{array} \right)$

$= \lim_{h\to 0} \frac{f(x+h) - f(x)}{x+h-x}$

$= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

= slope of tangent line at x.

Derivative Rules:

1) If $f(x) = c$, where c is a constant, then $f'(x) = 0$. **(Constant Function Rule)**

constant

2) If $f(x) = ax + b$, then $f'(x) = a$. **(Derivative of a Linear Function)**

3) If $f(x) = x^n$, where n is any nonzero real number, then $f'(x) = nx^{n-1}$. **(Power Rule)**

$y = t^{-3}$ $y' = -3 \cdot t^{-3-1} = -3 \cdot t^{-4}$

4) If $f(x) = ku(x)$, where k is a constant, then $f'(x) = ku'(x)$. **(Constant Multiple Rule)**

5) If $f(x) = u(x) + v(x)$, then $f'(x) = u'(x) + v'(x)$. If $f(x) = u(x) - v(x)$, then $f'(x) = u'(x) - v'(x)$. **(Sum and Difference Rules)**

Example: $y = \sqrt{7}$. Find y'. $= 0$ since it is constant.

① function
$f(x)$ or y

Example: $f(x) = x^5$. Find $f'(x)$. $= 5x^4$

(Newton) ② Derivative 1st notation.

$f'(x)$ or y'

⇒ Derivative of $f(x)$ 〃 y

Example: $y = t^{-3}$. Find $\frac{dy}{dt}$. $= -3 \cdot t^{-3-1} = -3t^{-4}$

variable = t

(Leibniz) ③ Derivative 2nd notation

$\frac{df(x)}{dx}$ or $\frac{dy}{dx}$

$\frac{d}{dx} \cdot f(x)$

Example: $\frac{d}{dx}\frac{1}{\sqrt[3]{x^2}}$. $= \frac{d\left(\frac{1}{\sqrt[3]{x^2}}\right)}{dx} = -\frac{2}{3} \cdot x^{-\frac{5}{3}}$

〃

$\frac{d}{dx}\frac{1}{(x^2)^{\frac{1}{3}}} = \frac{d}{dx}\left(\frac{1}{x^{\frac{2}{3}}}\right) = \boxed{\frac{d}{dx} x^{-\frac{2}{3}}} = -\frac{2}{3}x^{-\frac{2}{3}-1}$

$= -\frac{2}{3}x^{-\frac{5}{3}}$ (ex) $\frac{2}{3} = \frac{1}{3} \cdot 2$

Example: $f(x) = 3x^2$. Find $f'(x)$. $= 3 \cdot$ (derivative of x^2) $= 3 \cdot 2x = 6x$.

$x^2 \longrightarrow 2x$

Example: $y = 2x$. Find y'. $= 2\cdot$ (derivative of x) $= 2\cdot 1 = 2$.

$2 \quad x \longmapsto 1$

Example: $f(x) = 3x^2 + 7x - 9$. Find $f'(x)$.

$f'(x) = 6x + 7 - 0 = 6x + 7$.

Example: $y = \sqrt[3]{w} - 3w$. Find $\frac{dy}{dw}$.

① Change terms with exponent form

② Use power rule to each term

$y = w^{\frac{1}{3}} - 3w$

$\frac{dy}{dw} = \frac{1}{3} w^{-\frac{2}{3}} - 3$

$\frac{3}{5}x^2 \cdot \frac{1}{\sqrt{x}} = \frac{3}{5} \cdot x^2 \cdot \frac{1}{x^{\frac{1}{2}}} = \frac{3}{5} \cdot x^2 \cdot x^{-\frac{1}{2}}$

$= \frac{3}{5} \cdot x^{2-\frac{1}{2}}$

①

Example: $\frac{d}{dx}\frac{3x^2 + x^4}{5\sqrt{x}} = \frac{d}{dx}\left(\frac{3x^2}{5\sqrt{x}} + \frac{x^4}{5\sqrt{x}}\right)$

$x^{\frac{1}{2}}$

$= \frac{d}{dx}\left(\frac{3}{5} \cdot x^{2-\frac{1}{2}} + \frac{1}{5} x^{4-\frac{1}{2}}\right)$ ①

$= \frac{3}{5} \cdot (2-\frac{1}{2}) \cdot x^{2-\frac{1}{2}-1} + \frac{1}{5}(4-\frac{1}{2}) \cdot x^{4-\frac{1}{2}-1}$ ②

$= \frac{3}{5} \cdot \frac{3}{2} \cdot x^{\frac{1}{2}} + \frac{1}{5} \cdot (\frac{7}{2}) \cdot x^{\frac{5}{2}}$

$= \frac{9}{10} x^{\frac{1}{2}} + \frac{7}{10} x^{\frac{5}{2}}$ Simplification.

Applications

Example: An object moves along the y axis (marked in feet) so that its position at time t (in seconds) is $s(t) = t^3 - 6t^2 + 9t$. Find

derivative of s

$s(t) = t^3 - 6t^2 + 9t$

a) The instantaneous velocity function v.

$v = s'(t) = 3t^2 - 12t + 9$

b) The velocity at $t = 2$ and $t = 5$ seconds.

$v(5) = 3 \cdot 25 - 12 \cdot 5 + 9 = 75 - 60 + 9 = 24$

$v(2) = 3 \cdot 4 - 12 \cdot 2 + 9 = 12 - 24 + 9 = -3$ ft/s

c) The time(s) when the velocity is 0 ft/s.

$\Rightarrow$ Find t such that $v(t) = 0$

$3t^2 - 12t + 9 = 0$

$t^2 - 4t + 3 = 0 \Rightarrow (t-1)(t-3) = 0$

$t = 1$ or 3

Example: Let $f(x) = x^4 - 6x^2 + 10$. Find

a) The equation of the tangent line at $x = 1$.

b) Find the values of x where the tangent line is horizontal.

Example: The total sales of a company (in millions of dollars) t months from now are given by $S(t) = 0.015t^4 + 0.4t^3 + 3.4t^2 + 10t - 3$. Find $S(4)$ and $S'(4)$. Then, **interpret** both results.