# 142_18a_3.3

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### 3.3 Supplement: The Derivative

**The Derivative** - For $y = f(x)$, we define the **derivative of $f$ at $x$**, denoted by $f'(x)$, to be

$\boxed{= \text{slope of tangent line at } x}$

*If $f'(x)$ exists for each $x$ in the open interval $(a,b)$, then $f$ is said to be **differentiable** over $(a,b)$.

**Interpretations of the Derivative** - The derivative of a function $f$ is a new function $f'$. The domain of $f'$ is a subset of the domain of $f$. The derivative has various applications and interpretations, including the following:

1. **Slope of the Tangent Line** or
2. **Instantaneous Rate of Change** or
3. **Instantaneous Velocity** or

**Four-step Process for Finding the Derivative $f'(x)$**

**Example:** Use the four-step process to find $f'(x)$ if $f(x) = \sqrt{x} + 2$, and then use your result to find the equation of the tangent line of $f$ at $x = 9$.

Step ① $f(x+h) - f(x)$

Step ② $x+h - x = h$

Step ③ : Simplify $\dfrac{f(x+h)-f(x)}{h}$

Step ④ : Take limit on $\lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$.

$f'(x) = \dfrac{1}{2\sqrt{x}}$

① $f(x+h) - f(x)$

$= \sqrt{x+h} \cancel{+2} - (\sqrt{x} \cancel{+2})$

$= \sqrt{x+h} - \sqrt{x}$

② $h$

③ $\dfrac{(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x})}{h\,(\sqrt{x+h} + \sqrt{x})}$

$= \dfrac{\cancel{x}+h \cancel{-x}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})} = \dfrac{1}{\sqrt{x+h} + \sqrt{x}}$

④ $\lim_{h\to 0} \dfrac{1}{\sqrt{x+h} + \sqrt{x}} = \dfrac{1}{2\sqrt{x}}$

* $(a+b)(a-b) = a^2 - b^2$

$128 = 2^7$

$16 \cdot 12 = 2^4 \cdot 4 \cdot 3$

$= 2^4 \cdot 2^2 \cdot 3$

$= 2^6 \cdot 3$

$= 2^4 \cdot 2^2 \cdot 3$
$= 2^6 \cdot 3$

**Example:** The height of a ball thrown upward is given by $s(t) = -16t^2 + 128t$ feet, where $t$ is time in seconds. Use the limit definition of the derivative (i.e. the four-step process) to find the instantaneous velocity (i.e., velocity) when $t = 6$.

① $s(6+h) - s(6) = -16(6+h)^2 + 128\cdot(6+h) - (-16\cdot 6^2 + 128\cdot 6)$

$= -16(\cancel{36} + 12h + h^2) + \cancel{128\cdot 6} + 128\cdot h + \cancel{16\cdot 6^2} - \cancel{128\cdot 6}$

$= -16\cdot 12h - 16\cdot h^2 + 128\cdot h = \boxed{-64h - 16h^2}$

$128 - 16\cdot 12$

$= 2^7 - 2^6\cdot 3 = 2^6(2-3) = -2^6 = -64$

② $6+h-6 = h$.

③ $\dfrac{-64\cancel{h} - 16h^{\cancel{2}}}{\cancel{h}} = -64 - 16h$

④ $\lim_{h\to 0}(-64-16h) = -64$. feet/sec

**Nonexistence of the Derivative** - The existence of a derivative at $x = a$ depends on the existence of a limit at $x = a$, that is, on the existence of

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

If the limit does not exist at $x = a$, we say the function $\boxed{f \text{ is } \textbf{nondifferentiable at } x = a}$, or $\boxed{f'(a) \textbf{ does not exist}}$.

*Where does the above limit not exist (i.e. in what ways can a function $f$ fail to be differentiable)?

$s(t) = -t^2 + 128t$

① $s(t+h) - s(t) = -(t+h)^2 + 128(t+h) - (-t^2 + 128t)$

$= -(\cancel{t^2} + 2th + h^2) + \cancel{128t} + 128h + \cancel{t^2} - \cancel{128t}$

$= -2th - h^2 + 128h$

$= h(-2t - h + 128)$

② $t+h-t = h$

③ $\lim_{h\to 0} \dfrac{s(t+h) - s(t)}{t+h-t} = \lim_{h\to 0} \dfrac{\cancel{h}(-2t-h+128)}{\cancel{h}} = \lim_{h\to 0} -2t - h + 128$
$= -2t + 128$

*Note: If $f$ is differentiable at $a$, then $f$ is continuous at $a$. But, if $f$ is continuous at $a$, then $f$ is not necessarily differentiable at $a$.*

$s'(t) = -2t + 128$

Since linear equation has domain $\mathbb{R}$, $s$ is differentiable at everywhere

* $f(x)$ is differentiable at $x = a$
$\Leftrightarrow$ $a$ is inside of $\boxed{\text{domain of } f'(x)}$

**Sketching $\boxed{f' \text{ from } f}$:**

Observe the important points and general behavior of the original graph:

1) Points at which a tangent line is horizontal  $\boxed{\text{when } f'(x) = 0}$

2) Intervals over which the graph is increasing or decreasing

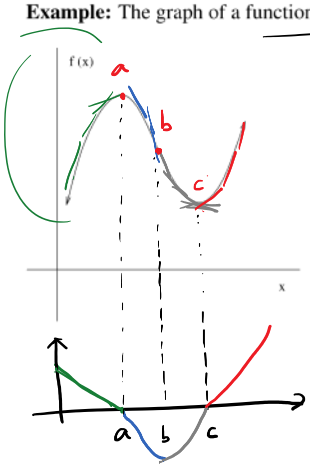
$f'(x) > 0 \Rightarrow$ graph is increasing (f(x))

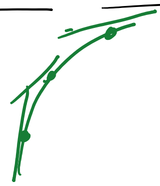
$f'(x) < 0 \Rightarrow f(x)$ is decreasing

3) Inflection points  These!

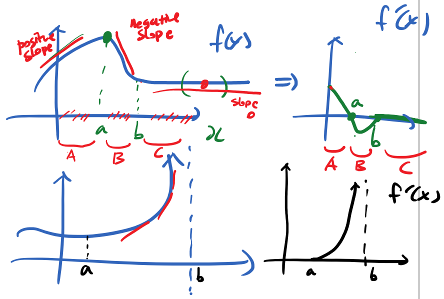
4) Places at which the graph appears to be horizontal or leveling off

**Example:** The graph of a function $f$ is given below. Sketch the graph of $f'$.

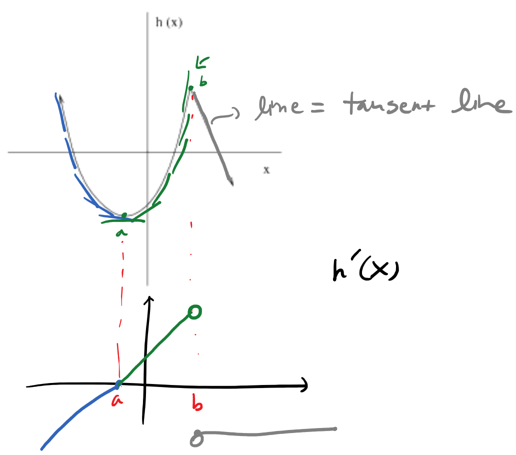


$f'(x)$




**Example:** Sketch the derivative of the function shown below.




$h'(x)$