Brief Pre-Calculus Review Notes (for class)

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Math 142 - Brief Pre-Calculus Review Notes

This review is to be a very quick refresher on concepts that are needed as incoming knowledge to succeed in this class. The goal is for it to motivate you to further review these concepts ASAP. To get a more thorough review, I strongly recommend that all students work through the Pre-Calculus Review Problem Set. As you work through those problems, if you are struggling with a particular area, you can read through the Completed Full Pre-Calculus Review Notes. All of these resources can be found on our eCampus course page under "Pre-Calculus Review Notes and Resources".

I. Functions

- Polynomial

exponent is non negative integer

$f(x) = 5 = 5 \cdot x^0$

$f(x) = 8x + 1$ (poly)

$= 8x^1 + 1 \cdot x^0$

$f(x) = 8x^{\frac{1}{2}}$ → not integer

$= 8\sqrt{x}$ (not poly)

$f(x) = x^2 + x + 1$ (poly)

- Rational

① $\frac{\text{poly}}{\text{poly}}$

$f(x) = \frac{x^2+1}{2x-2}$

② exponent is negative integer

$f(x) = x^{-1} = \frac{1}{x}$

$x^{\frac{1}{3}} = \sqrt[3]{x}$

$f(x) = \frac{\sqrt{x}}{2} = \frac{1}{2} \cdot x^{\frac{1}{2}}$ → not integer

(not poly not rational)

- Power

* one term function

$f(x) = \sqrt[2]{x} = \sqrt{x}$

$f(x) = 3x^{\frac{1}{4}}$

$f(x) = 2 \cdot x^{-1}$

x to the something (integer, rational, irrational... anything)

- Exponential

$f(x) = n^x$ n: any number.

$f(x) = 2^x$

$f(x) = e^x$

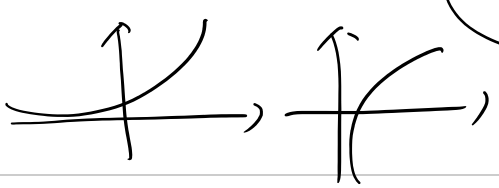
- Logarithmic

How much/many exponent is needed to get $n^a = x$

$f(x) = \log_n x$ (n: any positive number)

$f(x) = \log_2 x$

$\log_2 2^8 = 8 \log_2 2 = 8 \cdot 1 = 8$



II. Domain

- **Rules:**

① Denominator : $\neq 0$

② in case of $\sqrt[n]{x}$, n is even. $\Rightarrow x \geq 0$.

③ $\log_n x \Rightarrow x > 0$

$\log_2 x = 1$ ⟺ $2^1 = x$

$f(x) = \dfrac{\sqrt{x} - 2}{\log_3 x - 1}$: $x \neq 2$, $x \geq 0$, $x > 0$ ⇒ $x > 0, x \neq 2$ ⇒ $(0,2) \cup (2,\infty)$

Example 1: Find the domain of $f(x) = \dfrac{(x-3)\sqrt[4]{x-2}}{x^2+x-12}$. Write your answer using interval notation.

$\neq \dfrac{\sqrt[4]{x-2}}{x+4}$

When $x^2+x-12=0$? factor: $x \quad -3$, $x \quad 4$ ⇒ $(x-3)(x+4) = x^2+x-12 = 0$ ②

$x = 3, -4$.

⇒ x cannot be $3, -4$ because denominator goes 0.

⇒ $x \geq 2, x \neq 3$ $[2,3) \cup (3,\infty)$

⇒ ① $x \geq 2$ ⇐ $x - 2 \geq 0$

Example 2: Find the domain of $f(x) = \dfrac{\sqrt[5]{3x^2+4x-2}}{e^{x-9}}$. Write your answer using interval notation.

e^{x-9}

domain : $(-\infty, \infty)$

$(\frac{1}{2}, 9) \cup (9, 10)$

$n^{\log_n x} = x$

Example 3: Find the domain of $f(x) = \dfrac{\log_5(2x-1)}{\ln(10-x)}$. Write your answer using interval notation.

① $2x - 1 > 0$

$\ln(10-x) =: \log_e(10-x)$

① : $2x - 1 > 0$ ⇒ $2x > 1$ $x > \frac{1}{2}$

② $10 > x$

② $10 - x > 0$

③ $\ln(10-x) \neq 0$

$\ln(10-x) = 0$

$1 = e^0 = e^{\ln(10-x)} = 10 - x$

$x = 10 - 1 = 9$

③ $x \neq 9$ ⇐

(3) $x \neq 9$ $\Leftarrow$ $[\, 1 = e^0 = e^{\ldots} = 10 - x \,]$

2 $x = 10 - 1 = 9$

III. Piecewise-Defined Functions

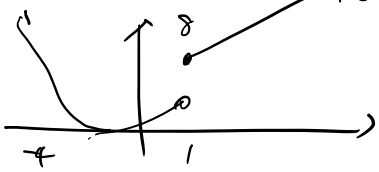
Definition: A **piecewise-defined function** is a function that is defined by different rules for different parts of its domain.

Example 4: Let

$$f(x) = \begin{cases} (x+1)^2 & -4 \le x < 1 \\ -x+9 & x \ge 1 \end{cases}$$

a) Find $f(-2)$ and $f(1)$.

b) Graph $f(x)$.



Example 5: Find the domain of the following function:

$$f(x) = \begin{cases} \dfrac{\sqrt[3]{x^2+4x-1}}{x^2-1} & 0 \le x < 2 \\ \dfrac{e^{12x-7}}{\log_2(5-x)} & x > 2 \end{cases}$$

Example 6: Write $f(x) = |x-8|$ as a piecewise-defined function.

Brief Pre-Calculus Review Highly Suggested Homework Problems: It is highly recommended that you attempt to work through the Pre-Calculus Review Problems that can be found under "Pre-Calculus Review Notes and Resources" on our course page in eCampus.