Math 142 Fall 2019 - Weak 3 - Review-Midterm
2/5/2020 -

Here's what you need to know to get the perfect grade.

(1) 5.6 Optimization: Be familiar with these two problems.

(a) Example: Suzie can sell 20 bracelets each day when the price is $10 for a bracelet. If she raises the price by $1, then she sells 2 fewer bracelets each day. If it costs $8 to make each bracelet, find the selling price that will maximize Suzies profit.

(b) Example: Ben needs to enclose two rectangular regions that share one side, and each has an area of 1400 ft . One of the regions needs to be enclosed on all four sides by a brick wall, and the other region just needs fencing on its remaining three sides (see diagram below). The brick wall costs $18 per foot, and the fencing costs $6 per foot. Find the dimensions of each region that would be the most economical for Ben.

(2) 6.1 Antiderivatives

(a) ***Antiderivative*** of a function $f(x)$: a function $F(x)$ such that $F'(x) = f(x)$.

(b) ***Indefinite integral:*** $\int f(x)dx := F(x) + C$ such that C is constant, F is an antiderivative.

(c) Properties of indefinite integral: For constant C and k,

(i) $\int x^n dx = \frac{1}{n+1}x^{n+1} + C$, where $n \neq -1$

(ii) $\int k dx = kx + C$

(iii) $\int e^x dx = e^x + C$

(iv) $\int \frac{1}{x} dx = \ln|x| + C$, where $x \neq 0$

(v) $\int kf(x)dx = k \int f(x)dx$

(vi) $\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$

(d) Example2, i): Find an indefinite integral $\int \frac{4u+u^3-3u^{-7}}{5u^2} du$

(e) Example4: The marginal revenue of selling x watches each day is given by $R'(x) = 30 - 0.0003x^2$ dollars per watch for $0 \leq x \leq 540$. If the revenue is \$1487.50 when 50 watches are sold, find the revenue function.

(f) Also see Example 5 in the previous lecture notes.

(3) 6.2: Substitution

(a) Reversing the chain rule!

(b) Example 1: $\int e^{x^3-1} \cdot 3x^2 dx$

(c) General Indefinite Integral Formulas

(i) $\int (f(x))^n \cdot f'(x)dx = \frac{(f(x))^{n+1}}{n+1} + C, n \neq -1$

(ii) $\int e^{f(x)} \cdot f'(x)dx = e^{f(x)} + C$

(iii) $\int \frac{1}{f(x)} \cdot f'(x)dx = \ln|f(x)| + C$

(d) Example 3 c): $\int \frac{2e^{5/x^4}}{3x^5} dx$

(e) Example 3 d): $\int \frac{8t^3}{\sqrt[7]{2-5t^4}} dt$

(4) 6.3: Estimating Distance Traveled

(a) We will estimate the area under a curve from $x = a$ to $x = b$ by dividing the region into subintervals (rectangles) of equal width.

$$\text{width of each subinterval } = \Delta x = \frac{b-a}{n}$$

where n is the number of subintervals (rectangles).

(b) (Left Sum) Example1: For the function $f(x) = 0.3x^2 + 2$, estimate the area of the region that lies under the graph of $f(x)$ between $x = -2$ to $x = 4$ using a left-hand sum with six subintervals of equal width.

(c) Example 6: The table below shows the velocity (ft/s) of an object every five seconds over a 20 second time interval. Estimate the total distance the object travels over the 20 second time interval by finding upper and lower estimates (i.e., right and left sums).

Time (s)	0	5	10	15	20
Velocity (fts)	25	31	35	43	47

(5) 6.4: The Definite Integral

(a) In general, we can use any x -coordinate, x_i^*, to find the the height of the rectangle in the i^{th} subinterval. Using summation notation, we can write the sum of the areas of the rectangles as

$$f\left(x_1^*\right)\Delta x + f\left(x_2^*\right)\Delta x + \cdots + f\left(x_n^*\right)\Delta x = \sum_{i=1}^{n} f\left(x_i^*\right)\Delta x$$

The sum $\sum_i^n f\left(x_i^*\right)\Delta x$ is called a Riemann sum.

(b) Then, the ***definite integral*** of $f(x)$ from a to b is $\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(x_i^*\right)\Delta x$

(c) Example 1: Use the graph of $f(x)$ below to find the following. Note that the graph consists of three straight lines and a semicircle.

(d) Example 3: Use a midpoint sum with $n = 3$ to estimate $\int_{-1}^{2} \left(x^2 - 1\right) dx$

(6) 6.5. The Fundamental Theorem of Calculus

(a) $\int_a^b f(x)dx$ gives an exact value and "counts" area above the x -axis positively and area below the x -axis negatively.

(b) Properties of Definite Integral:

(i) $\int_a^b m dx = m(b - a)$, where m is a constant

(ii) $\int_a^a f(x)dx = 0$

(iii) $\int_a^b f(x)dx = -\int_b^a f(x)dx$

(iv) $\int_a^b kf(x)dx = k\int_a^b f(x)dx$, where k is a constant

(v) $\int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$

(vi) $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$, where $a < c < b$

(c) Example 2: Use the graph of f (x) with the indicated areas below to answer the following.

(d) The Fundamental Theorem of Calculus, Part 2 - Suppose f is continuous on $[a, b]$.

$$\int_a^b f(x)dx = F(b) - F(a)$$

where F is any antiderivative of f, that is, $F' = f$

(e) Example 5: Evaluate $\int_2^K \left(t^2 + 4\right) dt$

(f) Example 6: A honeybee population starts with 200 honeybees and increases at a rate of $\left(n'(t) = 100e^{2t}\right)$ ees per week, where $\frac{t}{t}$ is in weeks and $t \geq 0$. a) Find the change in the honeybee population over the first 4 weeks. Round to the nearest integer, if necessary.

(g) Example 7: Consider the graph of $f'(x)$ shown below. If $f(0) = 50$, find $f(9)$.

(h) Average Value of a Continuous Function f over $[a, b]$

$$\frac{1}{b-a}\int_a^b f(x)dx$$

(7) 6.6: Area Between Two Curves

(a) Theorem: If $f(x)$ and $g(x)$ are two continuous functions with $f(x) \geq g(x)$ on $[a, b]$, then the area between the two curves on $[a, b]$ is given by

$$\int_a^b (f(x) - g(x))dx.$$

(b) Example 5: Find the area that is bounded by $y = -x^2$ and $y = 2x^3 - 5x$

(c) Example 7: Set up the definite integral(s) representing the area bounded by $y = -x^2 + 10x - 17$ and the x -axis on $[5, B]$, where $B > 8$