Math 142 Fall 2019 - Weak 3 - Review-Midterm
2/5/2020 -

Here's what you need to know to get the perfect grade.

(1) 4.3-4.4 Chain Rule.
  (a) Think a complicated function as a composition of simple functions.
  (b) For given $f(x) = g(h(x))$,
$$g'(h(x))h'(x) = f'(x) = \frac{df}{dx} = \frac{dg}{du} \cdot \frac{dh}{dx}.$$
  In other words, Leibniz Notation and Newton notation are just the same thing.
  (c) Ex) $f(x) = \log(5^{x^2-1})$. Find $f'(x)$.

(2) 5.1 The First Derivative
  (a) $f'(x) > 0$ (positive) on an interval $\iff$ f is increasing on interval.
  (b) $f'(x) < 0$ (negative) on an interval $\iff$ f is decreasing on interval.
  (c) $f(x)$ has a *local maxima* at $x = c$ if $f(c) \geq f(x)$ when x is near c.
  (d) $f(x)$ has a *local minima* at $x = c$ if $f(c) \leq f(x)$ when x is near c.
  (e) $f(x)$ has a *local extrema* at $x = c$ if f has local maxima or minima at $x = c$.
  (f) $f(x)$ has a *critical value* at $x = c$ if 1) $f(c)$ exists 2) $f'(c) = 0$ or does not exist.
  (g) First derivative test
  - Assumption: $f(x)$ is a continuous function on an interval (a, b), and $c \in (a, b)$ is a critical value of f.
  - Conclusion 1: If sign of $f'(x)$ changes from + (positive) to - (negative) at $x = c$, then f has a local maxima at c.
  - Conclusion 2: If sign of $f'(x)$ changes from - (negative) to + (positive) at $x = c$, then f has a local minima at c.
  - Conclusion 3: If sign of $f'(x)$ does not change at $x = c$, then f has no local minima or local maxima at c.

  (h) Ex) Find local extremas of $f(x) = 8 \ln x - x^2$ using the first derivative test.

(3) 5.2 The Second Derivative
  (a) $f''(x) = \frac{d^2 f}{dx^2}$; this is just notation.
  (b) f is *concave upward* on an interval (a, b) $\iff$ $f'(x)$ is increasing on (a, b) $\iff$ $f''(x) > 0$ (positive) on an interval (a, b).
  (c) f is *concave downward* on an interval (a, b) $\iff$ $f'(x)$ is decreasing on (a, b) $\iff$ $f''(x) < 0$ (negative) on an interval (a, b).
  (d) $f(x)$ has an *inflection point* at $x = c$ if 1) $f(c)$ exists 2) $f''(c) = 0$ or does not exist 3) sign of $f''(x)$ changes.
  (e) Second derivative test
  - Assumption: $f(x)$ is a continuous function on an interval (a, b), and $c \in (a, b)$ is a critical value of f such that $f'(c) = 0$.

- Conclusion 1: If $f''(x) > 0$ (positive), then f has a local minima at c.
- Conclusion 1: If $f''(x) < 0$ (negative), then f has a local maxima at c.
- Note: If c is a critical value but $f'(c)$ does not exist, then we cannot use the second derivative test.

(f) Ex) Find local extremas of $f(x) = 1 + 9x + 3x^2 - x^3$ using Second Derivative Test.

(4) 5.3 Limits at infinity.

(a) Given a curve $y = f(x)$, *vertical asymptote* is a line $x = a$ if $\lim_{x\to a^-} f(x) = \pm\infty$ or $\lim_{x\to a^+} f(x) = \pm\infty$.

(b) Difference between hole and vertical asymptote. ex) $f(x) = \frac{(x+1)(x-1)}{(x+1)(x-2)} \implies$ hole at $x = -1$, $VA = 2$.

(c) Given a curve $y = f(x)$, *horizontal asymptote* is a line $y = L$ if $\lim_{x\to\infty} f(x) = L$ or $\lim_{x\to-\infty} f(x) = L$. (So there are at most two horizontal asymptotes.)

(d) Find horizontal asymptote for rational function: Divide every term by the highest degree part of x in *denominator*. For example,

$$\lim_{x\to\infty} \frac{2x^2+1}{x^4+x-2} = \lim_{x\to\infty} \frac{2x^{-2}+x^{-4}}{1+x^{-3}-2x^{-4}} = \lim_{x\to\infty} \frac{0+0}{1+0-0} = 0.$$

(e) Find horizontal asymptote for rational function containing exponential term: Divide every term by the term of e^x which is *furthest* from zero. For example,

$$\lim_{x\to\infty} \frac{4e^{2x}-6e^{-3x}}{7+e^{2x}+e^{-6x}+e^{-8x}} = \lim_{x\to\infty} \frac{4-6e^{-5x}}{7e^{-2x}+1+e^{-8x}+e^{-10x}} = \frac{4-0}{0+1+0+0} = 4.$$

$$\lim_{x\to-\infty} \frac{4e^{2x}-6e^{-3x}}{7+e^{2x}+e^{-6x}+e^{-8x}} = \lim_{x\to-\infty} \frac{4e^{10x}-6e^{5x}}{7e^{8x}+e^{10x}+e^{2x}+1} = \frac{0-0}{0+0+0+1} = 0.$$

(5) 5.4 Graph Sketching

(a) Remember the table.

	$f'(x) > 0$	$f'(x) < 0$
$f''(x) > 0$	Increasing Concave Up	Decreasing Concave Up
$f''(x) < 0$	Increasing Concave Down	Decreasing Concave Down

(b) Critical points give local maxima or local minima generally.

(c) Use information of vertical and horizontal asymptote.

(d) Ex) Draw a graph from given information.

- Domain of $f : (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$
- $f(-2) = 1, f(0) = 0$, and $f(2) = 1$
- $f'(x) > 0$ on $(-\infty, -1)$ and (0,1)
- $f'(x) < 0$ on (-1,0) and $(1, \infty)$
- $f''(x) > 0$ on $(-\infty, -1), (-1, 1)$, and $(1, \infty)$
- Vertical asymptotes at $x = -1$ and $x = 1$
- $\lim_{x\to\infty} f(x) = 0$ and $\lim_{x\to-\infty} f(x) = 0$

(6) 5.5 Absolute Maxima and minima

(a) Assumption: $f(x)$ is a continuous function on a *closed* interval $[a, b]$. Let $c \in [a, b]$

(b) Conclusion 1: $f(x)$ has the *absolute maxima* at $x = c$ if $f(c) \geq f(x)$ for all $x \in [a, b]$

(c) Conclusion 1: $f(x)$ has the *absolute minima* at $x = c$ if $f(c) \leq f(x)$ for all $x \in [a, b]$

(d) How to find:

(i) Find all critical values of $f(x)$ inside of $[a, b]$.

(ii) Compare $f(x)$ at $x = a$, $x = b$, and $x =$ critical values. Find x giving the greatest (resp. the lowest) $f(x)$ among $x = a$, $x = b$, and $x =$ critical values. That x is the absolute maxima (resp. minima).

(iii) Ex) $f(x) = x^3 - 3x + 5$. Find the absolute maxima and absolute minima on $[0, 3]$.