# Section 6.2: Substitution

## Reversing the Chain Rule

Recall that in order to find the derivative of a composite function, we can use the Chain Rule:

$$\frac{d}{dx}\left(f\left(g(x)\right)\right) = f'\left(g(x)\right)\cdot g'(x)$$

In this section, we will look at reversing the Chain Rule in order to calculate an indefinite integral in which the integrand involves a Chain Rule. In other words,

$$\int f'\left(g(x)\right)\cdot g'(x)\,dx = f\left(g(x)\right) + C$$

**Example 1:** Evaluate the following integral:

$\int e^{x^3-1}\cdot 3x^2\,dx$

## General Indefinite Integral Formulas

1. $\int (f(x))^n\cdot f'(x)\,dx = \frac{(f(x))^{n+1}}{n+1} + C, n \neq -1$
2. $\int e^{f(x)}\cdot f'(x)\,dx = e^{f(x)} + C$
3. $\int \frac{1}{f(x)}\cdot f'(x)\,dx = \ln|f(x)| + C$

When it is not easy to recognize that the integrand is the result (or nearly the result) of the Chain Rule, the process of **substitution**, sometimes called ***u*-substitution**, can be used.

## Substitution Method:

1. Select $u$ (look for a more involved function where in the previous section we had just an $x$).
2. Take the derivative of $u$ using the $\frac{du}{dx}$ notation.
3. Bring $dx$ to the right hand side of the equation.
4. Bring any constant multiples to the left-hand side of the equation.
5. Substitute to replace everything involving $x$ in the integral. You should now have a basic integral involving $u$'s.
6. Integrate (i.e., find the anti-derivative) your basic integral involving $u$'s.
7. In your result, replace all $u$'s with $x$'s. This is your answer!

**Example 2:** Let's use $u$-substitution for the integral from Example 1.

$\int e^{x^3-1}\cdot 3x^2\,dx$

**Example 3:** Evaluate the following integrals:

a) $\int 7x(8x^2+3)^9\,dx$

b) $\int \frac{12x}{3x^2+5}\,dx$

c) $\int \frac{2e^{5/x^4}}{3x^5}\,dx$

d) $\int \frac{8t^3}{\sqrt[7]{2-5t^4}}\,dt$

e) $\int \frac{1}{x\ln x}\,dx$

f) $\int \frac{2e^{8x}}{e^{8x}-7}\,dx$

g) $\int \left(3x^4+6\right)\left(10x+x^5+7\right)^6\,dx$

h) $\int \left(25+4e^{5x-7}\right)\,dx$

**Example 4:** Find $f(x)$ if $f(2)=0$ and $f'(x)=\frac{x^4}{x^5+1}$.