Section 6.1: Antiderivatives

Antidifferentiation - Reconstructing a function from its derivative.

*A function F is an **antiderivative** of a function f if $F'(x) = f(x)$.

Example 1: Find three antiderivatives of $f(x) = x$.

*Thus, antidifferentiation leads not to a unique function, but to an entire **family** of functions (antiderivatives).

Indefinite Integrals - We use the symbol

$$\int f(x)\,dx$$

called the **indefinite integral**, to represent the family of antiderivatives of $f(x)$, and we write

$$\int f(x)\,dx = F(x) + C$$

if $F'(x) = f(x)$.

*The symbol $\int$ is called an **integral sign**, and the function $f(x)$ is called the **integrand**. The symbol dx indicates that antidifferentiation is performed with respect to the variable x. $F(x)$ is the antiderivative of $f(x)$, and the arbitrary constant C is called the **constant of integration**.

*Thus, for the above example, we could write $\int x\,dx = \frac{1}{2}x^2 + C$.

Formulas and Properties for Integration - For constants C and k,

1. $\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C$, where $n \neq -1$
2. $\int k\,dx = kx + C$
3. $\int e^x\,dx = e^x + C$
4. $\int \frac{1}{x}\,dx = \ln|x| + C$, where $x \neq 0$
5. $\int kf(x)\,dx = k\int f(x)\,dx$
6. $\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$

Example 2: Find the following indefinite integrals (i.e., the most general antiderivative)

a) $\int 8 \, dx$

b) $\int x^5 \, dx$

c) $\int \frac{1}{3}x^4 \, dx$

d) $\int \frac{3}{5x^2} \, dx$

e) $\int \left(5x^4 + x^3 - 2\right) \, dx$

f) $\int \frac{3}{x} \, dx$

g) $\int \left(3\sqrt{x} - \frac{1}{x^2} - x^{3/2} + 4e^x\right) \, dx$

h) $\int \left(\frac{6}{x} + \frac{5}{2x^4} - \frac{x^6}{3} \right) \, dx$

i) $\int \frac{4u + u^3 - 3u^{-7}}{5u^2} \, du$

j) $\int x \left(x^2 + 2 \right) \, dx$

k) $\int (x-2)(x+3) \, dx$

l) $\int \frac{9 - e^{-x}}{2e^{-x}} \, dx$

Example 3: Find $y(t)$ if $y(1) = 1$ and $\dfrac{dy}{dt} = \dfrac{3}{t} + \dfrac{1}{t^2}$.

Example 4: The marginal revenue of selling x watches each day is given by $R'(x) = 30 - 0.0003x^2$ dollars per watch for $0 \le x \le 540$. If the revenue is \$1487.50 when 50 watches are sold, find the revenue function.

Example 5: A sculpture purchased by a museum for \$50,000 increases in value at a rate of $V'(t) = 100e^t$ dollars per year, where t is the time in years since the purchase. What will the sculpture be worth in 12 years?