5.4 Supplement: Additional Curve Sketching

Graphing Strategy

1. Analyze $f(x)$.
   Find the domain of f.
   Find the intercepts.
   Find the asymptotes.
2. Analyze $f'(x)$.
   Find the partition numbers of $f'(x)$.
   Find the critical values of f.
   Construct a sign chart for $f'(x)$ to determine where f is increasing/decreasing,
   and find any local/relative extrema of f.
3. Analyze $f''(x)$.
   Find the partition numbers of $f''(x)$.
   Construct a sign chart for $f''(x)$ to determine where f is concave upward/downward,
   and find any inflection points.
4. Sketch the graph of f. Create a **combined number line** that shows the "shapes" of the graph, then draw the graph using all the pertinent information you found in steps 1-3.

Example: Use the graphing strategy to analyze the function $f(x) = \dfrac{2x^2+11x+14}{x^2-4}$. State all pertinent information and sketch the graph of f.

example continued...

Example: Use the graphing strategy to analyze the function $f(x) = xe^{2x}$. State all pertinent information and sketch the graph of f.

Example: Use the given information below to sketch the graph of f. **You must include sign charts with pertinent information for both $f'(x)$ and $f''(x)$, as well as a combined number line.**

- Domain of f: $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$
- $f(-2)=1$, $f(0)=0$, and $f(2)=1$
- $f'(x)>0$ on $(-\infty,-1)$ and $(0,1)$
- $f'(x)<0$ on $(-1,0)$ and $(1,\infty)$
- $f''(x)>0$ on $(-\infty,-1)$, $(-1,1)$, and $(1,\infty)$
- Vertical asymptotes at $x=-1$ and $x=1$
- $\lim_{x\to\infty} f(x)=0$ and $\lim_{x\to-\infty} f(x)=0$

Practice: On your on paper, try using the graphing strategy to analyze the function $f(x)=\dfrac{\ln x}{x}$. State all pertinent information and sketch the graph of f.