## 5.1 Supplement: The First Derivative

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**Increasing and Decreasing Functions**

- If $f'(x) > 0$ on an interval, then $f$ is increasing on that interval.
- If $f'(x) < 0$ on an interval, then $f$ is decreasing on that interval.

Some examples include...

*To determine where $f$ is increasing/decreasing, we will create a ____________________________. What "important" $x$ values should we include on our sign chart? What do we call these $x$ values?

**Example:** Find where $f(x) = (1-x)^{1/3}$ is increasing/decreasing.

## Local Maximum and Minimum Values:

- A function $f$ has a ______________________________ at $c$ if $f(c) \geq f(x)$ when $x$ is near $c$.
- A function $f$ has a ______________________________ at $c$ if $f(c) \leq f(x)$ when $x$ is near $c$.
- The local maximum and and minimum values of $f$ are called the ______________________________ of $f$.
- Where can local extrema occur?

## Critical Values:

- If $f$ has a local max/min at $c$, then ______________________________.
- A **critical value** of a function $f$ is a number $c$ in the domain of $f$ such that either $f'(c) = 0$ or $f'(c)$ does not exist.
- If $f$ has a local max/min at $c$, then $c$ ______________________________.
- Is the converse of the above statement true?

**Question:** How do we find the critical values of a function $f$?

**The First Derivative Test** - Suppose that $c$ is a number in the interval $(a,b)$ and $f$ is continuous on the interval $(a,b)$ . Also, let $c$ be a critical value of $f$.

- If $f'$ changes from positive to negative at $c$, then $f$ has a ______________________________ at $c$.
- If $f'$ changes from negative to positive at $c$, then $f$ has a ______________________________ at $c$.
- If $f'$ does not change sign at $c$, then $f$ has no local maximum or minimum at $c$.

**Example:** For each of the following, determine where the function is increasing/decreasing and find any local extrema.

a) $f(x) = x^3 - 6x^2 + 9x + 1$

b) $f(x) = \dfrac{1}{x-2}$

c) $f(x) = 8\ln x - x^2$

d) $f(x) = (x^2 - 3x - 4)^{4/3}$

e) $f(x) = (x+2)e^x$