

4.2 Supplement: Derivatives of Product and Quotients

The Product Rule - If $y = f(x) = F(x)S(x)$ and if $F'(x)$ and $S'(x)$ exist, then

$$f'(x) = F(x)S'(x) + S(x)F'(x)$$

Or, we could write...

Example: Find y' if $y = 2x^2(3x^4 - 5)$.

Example: Find $f'(x)$ if $f(x) = (x^2 + 3)(\sqrt[4]{x} + \sqrt[8]{x^3})$.

Example: Find $f'(t)$ if $f(t) = 10^t \log t$.

Example: Find $f'(x)$ if $f(x) = \pi x \log x^5$.

Example: Find y' if $y = \left(3w^2 + 4 \log_3 \left(\frac{6}{w^2}\right)\right) (2^w + 3e^w)$.

The Quotient Rule - If $y = f(x) = \frac{T(x)}{B(x)}$ and if $T'(x)$ and $B'(x)$ exist, then

$$f'(x) = \frac{B(x)T'(x) - T(x)B'(x)}{[B(x)]^2}$$

Or, we could write...

Example: Find $f'(x)$ if $f(x) = \frac{2\sqrt{x}}{x^2 - 3x + 1}$.

Example: $\frac{d}{du} \frac{4u^2 e^u}{\log_7 u + 5 \ln u}$.

Example; Find y' if $y = \frac{2x - 1}{(x^3 + 2)(x^2 - 3)}$.

Example: Find $\frac{dy}{dx}$ if $y = \frac{x^5 - 3x + 1}{23\sqrt[4]{x}}$.

Example: Find $f'(x)$ if $f(x) = \frac{e^2 - 4(3^x) - 3 \ln x}{4e^x - \sqrt{x} + \ln \pi + \log x}$.

Example: Suppose that the number x of DVD players a retail chain is willing to sell per week at a price of $\$p$ is given by

$$x = \frac{100p}{0.1p + 1}$$

where $10 \leq p \leq 70$.

a) Find dx/dp .

b) Find the supply and rate of change of supply when the price is \$40. Then, **interpret** your results.

c) Estimate the supply if price is increased to \$41 per DVD player.