4.1 Supplement: Derivatives of Powers, Exponents, and Sums

Derivative Notation - If $y = f(x)$, then

$$f'(x) \qquad y' \qquad \frac{dy}{dx} \qquad \frac{d}{dx}f(x)$$

all represent the derivative of f at x.

Derivative Rules:

1) If $f(x) = c$, where c is a constant, then $f'(x) = 0$. **(Constant Function Rule)**

2) If $f(x) = ax + b$, then $f'(x) = a$. **(Derivative of a Linear Function)**

3) If $f(x) = x^n$, where n is any nonzero real number, then $f'(x) = nx^{n-1}$. **(Power Rule)**

4) If $f(x) = ku(x)$, where k is a constant, then $f'(x) = ku'(x)$. **(Constant Multiple Rule)**

5) If $f(x) = u(x) + v(x)$, then $f'(x) = u'(x) + v'(x)$. If $f(x) = u(x) - v(x)$, then $f'(x) = u'(x) - v'(x)$. **(Sum and Difference Rules)**

Example: $y = \sqrt{7}$. Find y'.

Example: $f(x) = x^5$. Find $f'(x)$.

Example: $y = t^{-3}$. Find $\frac{dy}{dt}$.

Example: $\frac{d}{dx}\frac{1}{\sqrt[3]{x^2}}$.

Example: $f(x) = 3x^2$. Find $f'(x)$.

Example: $y = 2x$. Find $y\,'$.

Example: $f(x) = 3x^2 + 7x - 9$. Find $f\,'(x)$.

Example: $y = \sqrt[3]{w} - 3w$. Find $\dfrac{dy}{dw}$.

Example: $\dfrac{d}{dx}\dfrac{3x^2 + x^4}{5\sqrt{x}}$.

Applications

Example: An object moves along the y axis (marked in feet) so that its position at time t (in seconds) is $s(t) = t^3 - 6t^2 + 9t$. Find

a) The instantaneous velocity function v.

b) The velocity at $t = 2$ and $t = 5$ seconds.

c) The time(s) when the velocity is 0 ft/s.

Example: Let $f(x) = x^4 - 6x^2 + 10$. Find

a) The equation of the tangent line at $x = 1$.

b) Find the values of x where the tangent line is horizontal.

Example: The total sales of a company (in millions of dollars) t months from now are given by $S(t) = 0.015t^4 + 0.4t^3 + 3.4t^2 + 10t - 3$. Find $S(4)$ and $S'(4)$. Then, **interpret** both results.

Marginal Cost, Revenue, and Profit - If x is the number of units of a product produced in some time interval, then

1) **total cost** = $C(x)$ and **marginal cost** = $C'(x)$

2) **total revenue** = $R(x)$ and **marginal revenue** = $R'(x)$

3) **total profit** = $P(x) = R(x) - C(x)$ and **marginal profit** = $R'(x) - C'(x)$

*Marginal cost (or revenue or profit) is the instantaneous rate of change of cost (or revenue or profit) relative to production at a given production ______________.

Example: A company that makes grills has a total weekly cost function (in dollars) of $C(x) = 10,000 + 90x - 0.05x^2$, where x is the number of grills produced.

a) Find $C(500)$ and interpret your answer.

b) Find the marginal cost at a production level of 500 grills per week. Then, interpret your result.

c) Approximate/estimate the cost of producing 501 grills.

d) Approximate/estimate the cost of the 501st grill.

e) Find the **exact cost** of producing the 501st grill.

Derivatives of Exponential and Logarithmic Functions

1. $\frac{d}{dx}e^x = e^x$
2. $\frac{d}{dx}\ln x = \frac{1}{x}$
3. $\frac{d}{dx}b^x = b^x \ln b$
4. $\frac{d}{dx}\log_b x = \frac{1}{\ln b}\left(\frac{1}{x}\right)$

Example: Find $\frac{dy}{dx}$ for the following. **Do not simplify your answer!**

a) $y = \log_\pi x$

b) $y = 7.129^x$

c) $y = 8e^x - e^8 + \frac{1}{9}\ln x$

d) $y = x^4 - 4^x - 3\log_6 x + 7(5^x)$

e) $y = \frac{x^3 - 3x(2^x) - \frac{4}{11}}{x}$

Note: In some cases it might be necessary to simplify a function using logarithmic properties before taking the derivative.

Example: Find $f'(x)$ if $f(x) = 5 + 7\ln\frac{6}{x^3}$.

Example: Find the equation of the line tangent to the graph of $f(x) = 1 + \ln x^4$ at $x = e$.

Example: The price-demand equation of a store that sells x hats at a price of p dollars per hat is given by $p = 350(0.999)^x$. Find the rate of change of price with respect to demand when the demand is 800 hats. Then, interpret your result.