## 3.3 Supplement: The Derivative

**The Derivative** - For $y = f(x)$, we define the **derivative of** $f$ **at** $x$, denoted by $f'(x)$, to be

*If $f'(x)$ exists for each $x$ in the open interval $(a,b)$, then $f$ is said to be **differentiable** over $(a,b)$.

**Interpretations of the Derivative** - The derivative of a function $f$ is a new function $f'$. The domain of $f'$ is a subset of the domain of $f$. The derivative has various applications and interpretations, including the following:

1. **Slope of the Tangent Line** or
2. **Instantaneous Rate of Change** or
3. **Instantaneous Velocity** or

**Four-step Process for Finding the Derivative $f'(x)$**

**Example:** Use the four-step process to find $f'(x)$ if $f(x) = \sqrt{x} + 2$, and then use your result to find the equation of the tangent line of $f$ at $x = 9$.

**Example:** The height of a ball thrown upward is given by $s(t) = -16t^2 + 128t$ feet, where $t$ is time in seconds. Use the limit definition of the derivative (i.e. the four-step process) to find the instantaneous velocity (i.e., velocity) when $t = 6$.

<u>**Nonexistence of the Derivative**</u> - The existence of a derivative at $x = a$ depends on the existence of a limit at $x = a$, that is, on the existence of

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

If the limit does not exist at $x = a$, we say the function $f$ is **nondifferentiable at** $x = a$, or $f'(a)$ **does not exist**.

*Where does the above limit not exist (i.e. in what ways can a function $f$ fail to be differentiable)?

**Note: If $f$ is differentiable at a, then $f$ is continuous at a. But, if $f$ is continuous at a, then $f$ is not necessarily differentiable at a.*

**Sketching $f'$ from $f$:**

Observe the important points and general behavior of the original graph:

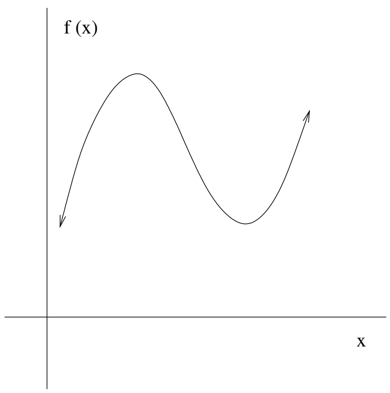
1) Points at which a tangent line is horizontal

2) Intervals over which the graph is increasing or decreasing

3) Inflection points

4) Places at which the graph appears to be horizontal or leveling off

**Example:** The graph of a function $f$ is given below. Sketch the graph of $f'$.




**Example:** Sketch the derivative of the function shown below.