3.2 Supplement: Rates of Change

Slope of the Secant Line/Average Rate of Change

*__Slope of the Secant Line__ - A line through two points on the graph of a function is called a **secant line**. If the points $(a, f(a))$ and $(a+h, f(a+h))$ are two points on the graph of $y = f(x)$, then the slope of the secant line is given by

*The slope of the secant line can also be interpreted as the _____ or _____. Some examples of the average rate of change include...

Slope of the Tangent Line/Instantaneous Rate of Change

*__Slope of the Tangent Line__ - Given $y = f(x)$, the **slope of the tangent line**, or **slope of the graph**, at the point $(a, f(a))$ is given by

NOTE: The above limit exists if and only if the slopes of the secant lines between $x = a$ and x values to the _____ and _____ of a approach the same value (i.e. the slope of the tangent line).

*The slope of the tangent line can also be interpreted as the _____ or _____. Some examples of the instantaneous rate of change include...

Example: The revenue (in dollars) from the sale of x toasters each week is given by

$$R(x) = 50x - 2x^2$$

where $0 \leq x \leq 25$.

a) Find the change in revenue if production increases from 7 to 15 toasters each week.

b) Find the average change in revenue if production increases from 7 to 15 toasters. Then, **interpret** your answer.

c) Find the rate of change of revenue at a production level of 7 toasters. Then, **interpret** your answer.

Example: Suppose an object moves along the y axis so that its location is $y = f(x) = x^2 + x$ at time x, where y is in meters and x is in seconds. Find

a) The average velocity between 2 and 4 seconds.

b) The average velocity between 2 and $2+h$ seconds.

c) The velocity at 2 seconds.

Example: Let $f(x) = 3x^2$ and find

a) The slope of the secant line between $x = 2$ and $x = 5$ (i.e. between the points $(2, f(2))$ and $(5, f(5))$).

b) The equation of the tangent line at $x = 2$ (i.e. at $(2, f(2))$).

Example: The following table gives some values of a function, $f(x)$, rounded to 5 decimal places. Use the information to estimate the slope of the tangent line to $y = f(x)$ at $x = 1$.

x	0.98	0.99	1	1.01	1.02
$f(x)$	0.98995	0.99499	1	1.00499	1.00995

Example: The table below gives values of $P(t)$, the population of a small city in Texas in year t. (Midyear estimates are given.)

t	1994	1996	1998	2000	2002
$P(t)$	29,036	29,672	32,300	36,205	38,260

Find the average rate of growth from 1996 to 2000, and interpret your answer. (Round your final answer to the nearest integer, if necessary.)